Goos-Hänchen shift in negatively refractive media

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The Goos-Hänchen shift is calculated when total internal reflection occurs at an interface between ''normal'' and negatively refractive media. The shift is negative, consistent with the direction of energy flow in the negatively refractive medium.

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There has been renewed interest in media having negative index of refraction [1,2]. A theory of refraction at an interface between right-handed ($\epsilon>0,\ \mu>0$) and left-handed or negative refractive index ($\epsilon<0,\ \mu<0$) media was developed by Veselago [3]. In this Brief Report, I would like to consider the Goos-Hänchen effect (a lateral displacement that a light beam experiences in total internal reflection) at such an interface. It will be seen that an interpretation of this effect in terms of an energy flow in the medium having a lower index of refraction [4] remains valid. Consistent with this interpretation, the Goos-Hänchen shift occurs in a direction *opposite* to that in total internal reflection between right-handed media. Negative Goos-Hänchen shifts also occur for media having negative ϵ and positive μ, but only for TM waves [5].

The appropriate field vectors are shown in Fig. 1, assuming that the electric-field vector is polarized perpendicular to the plane of incidence (TE waves). Medium 1 is right handed and medium 2 is left handed. The electric fields are taken as

$$\mathbf{E}(\mathbf{r},t)=\mathrm{Re}[\mathbf{E}\exp\{i(k_x x+k_y y-\omega t)\}]$$
$$=\mathrm{Re}[E\exp\{i(k_x x+k_y y-\omega t)\}]\hat{\mathbf{z}},$$

$$\mathbf{E}'(\mathbf{r},t)=\mathrm{Re}[\mathbf{E}'\exp\{i(k'_x x+k'_y y-\omega t)\}]$$
$$=\mathrm{Re}[E'\exp\{i(k'_x x+k'_y y-\omega t)\}]\hat{\mathbf{z}},$$

$$\mathbf{E}''(\mathbf{r},t)=\mathrm{Re}[\mathbf{E}''\exp\{i(k''_x x+k''_y y-\omega t)\}]$$
$$=\mathrm{Re}[E''\exp\{i(k''_x x+k''_y y-\omega t)\}]\hat{\mathbf{z}},$$

where

$$k_x^2+k_y^2=k_x''^2+k_y''^2=k_1^2=\epsilon_1\mu_1\omega^2,$$

$$k_x'^2+k_y'^2=k_2^2=\epsilon_2\mu_2\omega^2.$$

With the incident wave chosen such that

$$k_x=k_1\cos\theta,\quad k_y=k_1\sin\theta,\quad k_1>0,$$

all properties of the transmitted and reflected waves are determined from Maxwell's equations. The magnetic fields are given by

$$\mathbf{H}(\mathbf{r},t)=\mathrm{Re}[\mathbf{H}\exp\{i(k_x x+k_y y-\omega t)\}]$$
$$=\mathrm{Re}[E(\mu_1\omega)^{-1}(k_y\hat{\mathbf{x}}-k_x\hat{\mathbf{y}})\exp\{i(k_x x+k_y y-\omega t)\}],$$

$$\mathbf{H}'(\mathbf{r},t)=\mathrm{Re}[\mathbf{H}'\exp\{i(k'_x x+k'_y y-\omega t)\}]$$
$$=\mathrm{Re}[E'(\mu_2\omega)^{-1}(k'_y\hat{\mathbf{x}}-k'_x\hat{\mathbf{y}})$$
$$\times\exp\{i(k'_x x+k'_y y-\omega t)\}],$$

$$\mathbf{H}''(\mathbf{r},t)=\mathrm{Re}[\mathbf{H}''\exp\{i(k''_x x+k''_y y-\omega t)\}]$$
$$=\mathrm{Re}[E''(\mu_1\omega)^{-1}(k''_y\hat{\mathbf{x}}-k''_x\hat{\mathbf{y}})$$
$$\times\exp\{i(k''_x x+k''_y y-\omega t)\}].$$

The fields in Fig. 1 are drawn assuming that k'_x and k''_x are negative, while k'_y and k''_y are positive, with $\mu_1>0,\ \mu_2<0$. As will be seen, this is a consistent assignment for angles of incidence less than the critical angle. Extension to total internal reflection is then straightforward.

From the boundary conditions at $x=0$, it follows that

$$k'_y=k''_y=k_y,$$

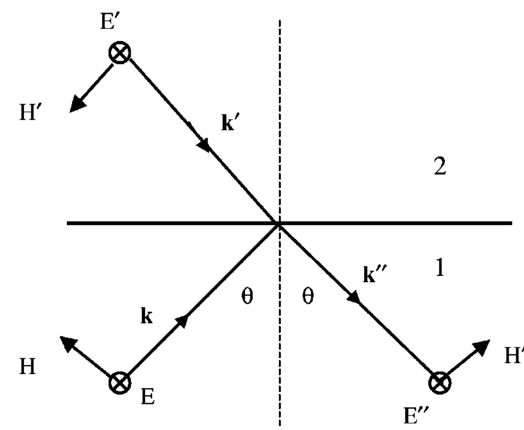

FIG. 1. The incident, reflected, and transmitted waves when medium 1 is right handed ($\epsilon_1>0,\ \mu_1>0$) and medium 2 is left handed ($\epsilon_2<0,\ \mu_2<0$), for angles of incidence less than the critical angle. In medium 2, the energy flow is in a direction opposite to $\mathbf{k}'$.

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which implies that $k'_x = \pm\sqrt{\epsilon_2\mu_2\omega^2 - k_1^2\sin^2\theta}$, $k''_x = \pm k_1\cos\theta$. The signs must be chosen to ensure energy flow away from the interface, namely

$$k'_x = -\sqrt{\epsilon_2\mu_2\omega^2 - k_1^2\sin^2\theta}, \quad k''_x = -k_1\cos\theta.$$

The energy flow in medium 2 is away from the interface since $\mathbf{E}'\times\mathbf{H}'$ is in the $-\mathbf{k}'$ direction. The reflection and transmission coefficients are obtained by requiring that the tangential components of both **E** and **H** be continuous at the boundary. If $(\epsilon_2\mu_2\omega^2 - k_1^2\sin^2\theta)>0$, one recovers the standard Fresnel formulas

$$\frac{E'}{E} = \frac{2\cos\theta}{\cos\theta + \sqrt{\frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}}\sqrt{1-\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta}},$$

$$\frac{E''}{E} = \frac{\cos\theta - \sqrt{\frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}}\sqrt{1-\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta}}{\cos\theta + \sqrt{\frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}}\sqrt{1-\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta}}.$$

For angles of incidence less than the critical angle, the Fresnel equations are identical for right- and left-handed media.

On the other hand, when $(\epsilon_2\mu_2\omega^2 - k_1^2\sin^2\theta)<0$, one must choose

$$k'_x = i\sqrt{k_1^2\sin^2\theta - \epsilon_2\mu_2\omega^2}$$

to ensure that the energy does not diverge for large positive x. The boundary conditions on the field vectors are

$$E + E'' = E',$$

$$\frac{k_x}{\mu_1}(E - E'') = \frac{k'_x}{\mu_2}E' = -i\frac{\sqrt{\epsilon_1\mu_1}}{|\mu_2|}\sqrt{\sin^2\theta - \frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}.$$

It then follows that the reflection amplitude $\sqrt{R} = |\sqrt{R}|e^{i\phi_R}$ is given by

$$\sqrt{R} = \frac{E''}{E} = -\frac{\cos\theta + i\alpha}{\cos\theta - i\alpha} = -\frac{k_x + i\left|\frac{\mu_1}{\mu_2}\right|\sqrt{k_1^2 - k_2^2 - k_x^2}}{k_x - i\left|\frac{\mu_1}{\mu_2}\right|\sqrt{k_1^2 - k_2^2 - k_x^2}},$$

where

$$\alpha = \left|\frac{\mu_1}{\mu_2}\right|\sqrt{\sin^2\theta - \frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}.$$

The corresponding formula for right-handed media is

$$\sqrt{R} = \frac{E''}{E} = \frac{\cos\theta - i\alpha}{\cos\theta + i\alpha} = \frac{k_x - i\left|\frac{\mu_1}{\mu_2}\right|\sqrt{k_1^2 - k_2^2 - k_x^2}}{k_x + i\left|\frac{\mu_1}{\mu_2}\right|\sqrt{k_1^2 - k_2^2 - k_x^2}}.$$

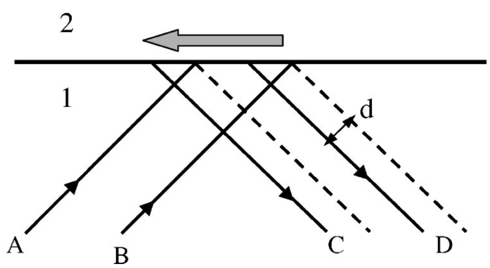

FIG. 2. A schematic representation of the Goos-Hänchen shift d. AB is the incident beam and CD the reflected beam. Medium 2 is negatively refractive with energy flow to the left.

Thus, the phase of the reflection coefficient has the same magnitude, but differs in *sign* for right- and left-handed media.

The Goos-Hänchen shift can be calculated in terms of the phase shift of the reflection coefficient in total internal reflection as $d = -k_1^{-1}\sin\theta d\phi_R/dk_x$ [6,7]. Since the phase shift for left-handed media is opposite to that for right-handed media, the Goos-Hänchen shift is to the *left* in Fig. 2. With the notation in Fig. 2, one finds a displacement

$$d = \frac{2}{k_1}\frac{|\mu_2/\mu_1|\sin\theta}{\left[(\mu_2/\mu_1)^2\cos^2\theta + \sin^2\theta - \frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}\right]\sqrt{\sin^2\theta - \frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}}.$$

This expression is not valid at the critical angle or for $\theta = \pi/2$, where $d\phi_R/dk_x$ is not defined [7]. For the **H** field polarized perpendicular to the plane of incidence (TM waves), one need only make the replacement $\mu_2/\mu_1 \leftrightarrow \epsilon_2/\epsilon_1$.

An alternative derivation of the shift uses the idea that energy is transferred from one side of the field to the other by propagating through medium 2 in the horizontal direction [4]. For right-handed media the energy would propagate to the right and the displacement of the beam is to the right. However, for left-handed media, the energy flow (opposite to k) is to the *left* and the direction of the Goos-Hänchen shift is reversed. Although the validity of the expression obtained by Renard [4] using this approach has been questioned [7], the qualitative picture of energy flow on which he based his calculation seems to be justified [5].

In summary, I have shown that the Goos-Hänchen shift for total internal reflection from a right-handed to a left-handed medium is in a direction opposite to that for total internal reflection between two right-handed media. Moreover, this result is consistent with the idea that energy is transported from one side of the incident beam to the other side by propagation through the rarer medium.

After submission of this paper, I came across a paper by Lakhtakia [8], in which a reversed Goos-Hänchen shift is obtained for negatively refracting, dissipative media. Following the submission of this paper, there appeared a paper [9] that puts into question whether or not effects such as negative refraction can be observed experimentally, given the fact that left-handed media are necessarily dispersive. The conclusions in that paper are consistent with the numerical calculations in Ref. [2] that found no evidence for perfect lensing by a slab of left-handed material. Thus, the possibility of observing a reversed Goos-Hänchen shift, calculated in this paper for ideal, nondispersive left-handed materials, remains an open question. Negative Goos-Hänchen shifts have also been predicted for reflection near the Brewster dip in weakly absorbing right-handed materials [10].

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